Equation and Nomogram for Calculation of Testing and Treatment Thresholds

To the Editor:—According to the threshold model, the choice of a particular clinical strategy, observation vs testing vs treatment, is dictated by the probability of disease’s exceeding two relevant threshold probabilities.* If the probability of disease is below the testing threshold \( (p_t) \), testing should be withheld. If the probability of disease is above the treatment threshold \( (p_a) \), then treatment should be given. The test should be performed only if the probability of disease is between the two thresholds. These threshold probabilities can be calculated from the data on the benefits and risks of the appropriate treatments, the sensitivity and specificity of a particular diagnostic test, and the risks associated with the test.

We further generalize this model by providing an equation for calculation of any threshold probability*:

\[
p_t = \frac{1}{LR + \frac{B}{R} + 1}
\]

where \( LR \) is the likelihood ratio, \( B \) is the benefit experienced by treated patients with the disease, and \( R \) is the risk experienced by treated patients without the disease. To calculate \( p_a \), use \( LR + (LR > 1) \); to calculate \( p_t \), use \( LR - (LR < 1) \). If \( LR = 1 \), then \( p_t = p_a \). The last possibility relates to a clinical situation when no further tests are available.' Equation 1 thus can be used for easy calculation of the threshold probability of interest.

An even more convenient way to represent a functional relationship among the variables in equation 1 \( (p_a, LR, B/R) \) is to construct a nomogram (Fig. 1). The threshold probability is read at the point of intersection of the line drawn through known values of \( B/R \) and \( LR \) on their corresponding axes. For example, the benefit-risk ratio for treatment with anticoagulants of a patient with suspected pulmonary embolism (PE) is 6.2. The positive and negative likelihood ratios of a ventilation-perfusion scan (V/Q) for diagnosis of PE are 16.8 and 0.17, respectively. Drawing lines through these points gives us \( p_a = 0.95\% \) and \( p_t = 49\% \). This means that if our estimate of the probability of PE is less than 0.95\% we should not order a V/Q scan; if suspicion that the patient has PE is greater than 49\% then we should administer anticoagulants without previously ordering a V/Q scan. A ventilation-perfusion scan should be ordered if suspicion for PE is in the range between 0.95\% and 49\%, notice that if a V/Q scan is not available, we can draw the line through \( LR = 1 \), giving us \( p_t = 14\% \).

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"Equation 1 was derived by assuming that the risks of ‘diagnostic tests are negligible with the benefits and the risks of treatment and by further substitution of expressions for test sensitivity and specificity in Pauker and Kassirer’s original threshold equations’ with their respective likelihood ratios \( (LR = sensitivity/false-positive rate; LR - = false-negative rate/specificity). Notice that original threshold equations should be used if diagnostic tests are associated with considerable risks.'
A similar nomogram for performing threshold analysis was developed by Glasziou. Glasziou's nomogram requires separate calculation of the treatment threshold probability when no diagnostic tests are available prior to allowing determination of two other thresholds. With this additional step, an intuitive feeling for the benefit-risk ratios of available treatments is lost. In our experience, Glasziou's nomogram has not enabled physicians to easily capture a central relationship between the benefit-risk ratios of available treatments and the threshold probabilities. Our nomogram permits reading threshold probabilities in direct relation to the benefit-risk ratio of the treatment under consideration.

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References